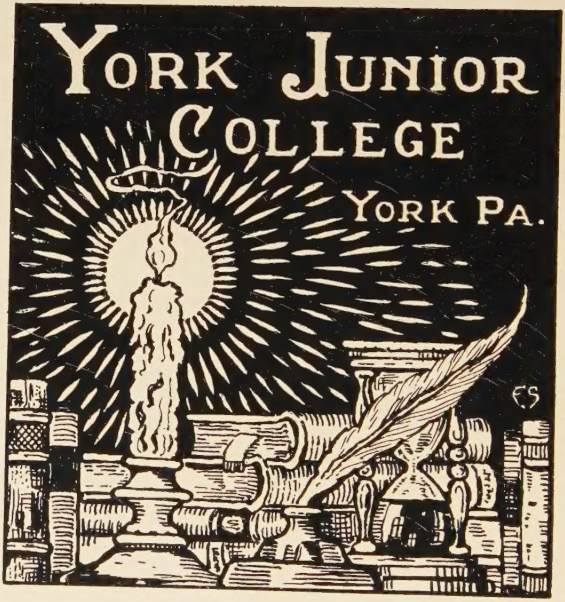
THE THEORY OF RELATIVITY

ERWIN FREUNDLICH



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Francis Forgan
April 1924

THE THEORY OF RELATIVITY

THREE LECTURES FOR CHEMISTS

BY

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METHUEN & CO. LTD.
36 ESSEX STREET W.C.
LONDON

This Translation First Published in 1924

PRINTED IN GREAT BRITAIN

QC
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INTRODUCTION

By VISCOUNT HALDANE

MR. BROSE, who has already rendered into our own language "The Foundations of Einstein's Theory of Gravitation," by Dr. Erwin Freundlich, has now given us an excellent translation of the recent lectures by the latter on "The Theory of Relativity."

Dr. Freundlich is one of the most brilliant of the younger astronomical adherents in Germany of Einstein, and he brings to the present exposition a firm grasp both of its physical and its mathematical principles. The great value of these lectures as a means of making the theory of Relativity in physics intelligible to the general reader is the way in which two of the fundamental features are brought into prominence. The first of these

is that there is a finite velocity which we are really compelled to presuppose as constant in observation, whatever may be our co-ordinates and whatever our system. This is the velocity of light. Observation shows that it is always found to be a constant, and must therefore be made so in our formulas, with a variation of form in the general theory which Einstein added to his earlier restricted theory. Newtonian physics left the velocity of light to be treated as infinite. The new doctrine treats it as the outcome of experiment and observation, as being a finite velocity which in the main plays the part of an upper limiting velocity by reference to which we always estimate. The quantities which we call length, volume, and duration have a meaning only within the finite limits imposed on us by observation.

The second feature brought into great prominence by Dr. Freundlich is that of the equality of inertial and gravitational energy. Centrifugal and gravitational mass become indistinguishable. Gravitational effects are in ultimate analysis distinguishable only as

states of acceleration, a mere auxiliary and derivative conception. *All* motion is only the motion of bodies relatively to one another, and this doctrine comes out in its full effect in the new general theory. The principle that is characteristic of all motion is for Einstein that every body moves under the influence of inertia and gravitation along the shortest and straightest path, which is a geodetic line in the variable space-time manifold. The surface here is by no means necessarily Euclidean. We have to turn back to Riemann's teaching to learn that it is only from experience that we can judge of the true character of actual space. The deviations from Euclidean measure-relations, as formed between points indefinitely close, which give rise to the curvilinear accelerated motions of bodies, may be measured in a wider geometry of the relations of indefinitely close points, with characteristics attributed to these relations which, in a geometry which must be one of the infinitely near, is independent of any choice of co-ordinates giving rise to distinctive shapes and measurements in finite spaces and times. The

result is that the laws of conservation of energy and mass which have hitherto been treated as independent in Newtonian physics, become examples of a single principle of conservation of energy. The analogy between moving energy particles and moving matter becomes complete, and the conception of electromagnetic mass, hitherto a puzzle, is now intelligible. Mechanics and electrodynamics become branches of a single science. For matter and energy are now identical in nature. Energy itself possesses inertia, and consequently increase in the energy of a body means increase in its mass. Another of the older views, that of mass as an independent fact, is thereby superseded. The length and volume of a body have ceased to have more than a relative meaning, depending on the conditions of observation, for instance, on whether the observer is at rest or in motion. The conception of mass is also deposed from its absolute position, and is identified with that of energy, which has, in common with what we call matter, the property of inertia. Matter is just a form under which energy

appears. Gravitation must be ascribed to it as the outcome of the discovery of the inertia of energy. But no effect, whether inertial or gravitational, can be propagated with a velocity greater than the limiting one of light. Electromagnetic mass is a form of the inertia of matter thus conceived. This explains some facts observed when electrons are produced in a cathode ray tube, facts which were obscure so long as the electrons had to be interpreted to be material particles instead of pure energy particles. Clerk Maxwell's equations have now a new application in the light of what Lorentz and Einstein have discovered. It is to the laws which govern the curvature of the space-time continuum that we are to look for final explanations of the behaviour of objects.

Such is the substance of what Dr. Freundlich explains with the same lucidity as was distinctive of his earlier book on the "Foundations of Einstein's Theory of Gravitation." The new exposition seems to me, untrained as mathematician, a very valuable one. The mathematical explanations are of a kind as simple as they are penetrating. A general

reader who will take the trouble can at least understand what he is told in simple language. He will probably, if he does, find himself as well off as will at least some mathematicians.

THE THEORY OF RELATIVITY

FIRST LECTURE

NEWTONIAN MECHANICS

IT has given me great pleasure to accept the invitation to present to you a report on Einstein's investigations concerning the general theory of relativity. For we are experiencing a phase of such exceeding importance in the development of natural science that every physicist, chemist, and engineer, whether he be occupied with the theory or with the practical applications of his subject, is constrained to know something of these new researches. I have accepted this invitation with the more pleasure on this occasion because, thanks to the ready help of Professor Bosch, it is here that I have found a favourable

centre in which to attack, in collaboration with Dr. Hochheim, a question which will be of great importance for testing the theory.

The characteristic feature of the stage of growth which modern physics is at present undergoing is that two great realms of research which had hitherto developed independently of each other were suddenly beset with problems which required a joint application of their laws. It hereby came to light that the old views did not allow these two realms to be combined into one physical world-picture free from inner contradictions. It was only the advent of the theory of relativity that allowed this goal to be achieved. To give a just estimate of what this has accomplished, I shall endeavour to give in three successive lectures at least an introduction into the underlying ideas of this theory, and I hope to be able to give you a glimpse of the creative genius of Einstein, who, by using scarcely more facts of experience than those already known to Galilei and Newton, has found a way of building up an entirely new and uniformly consistent physical picture of the world. Naturally, opposition to these new ideas will not be lacking in the

future, so long as they strive after universal validity, and this is but human. For, at first sight, many will not readily overcome the feeling of disappointment at the circumstance that we have once again had to experience a thorough convulsion of the foundations of physics. For were we not all convinced and did we not all gain the impression in our student days that the foundations of physics were so firmly established that such a convulsion was no longer possible? I hope to be able to show—by devoting this first lecture to a short sketch of existing physics up to the special theory of relativity—that, in reality, the foundations of our earlier physics have hitherto been quite incomprehensible and enigmatic, and that we should always have expected them to be revised in the manner now before us. But you will then also see that this revision could have come about only now that, on the one hand, our methods of research have been sufficiently refined in certain respects and, on the other hand, a sufficient number of new facts of experience have been found which have prepared the way for the new views.

What we here call the physical world-picture

comprehends two great complexes of phenomena :

(1) *The phenomena of motion of bodies perceptible as changes of position,* and

(2) *The electromagnetic phenomena in and between bodies* (*electrodynamics*).

Only a few decades ago, indeed, so simple a division would not have been possible. The phenomena of light, of sound, of the electric current, and so forth, each claimed to be a region which required a special discipline. We regard it as a very marked advance in the development of physics that we have succeeded in compressing the rich abundance of phenomena into two great complexes, namely, into phenomena of motion and into electromagnetic phenomena of matter.

The object of mechanics is the study of motional events. In it, a fundamental conception rules supreme in all events : the conception of *mass*. In electrodynamics, the object of which is the study of all electric and magnetic phenomena, its place is taken by the conception of *energy*. It is very much more difficult to illustrate the latter because we have not so simple a sense as that of touch

at our disposal to do so. We were, however, compelled to form this conception to obtain a measure of the quantity that we call *work*, and which manifests itself in very different forms, as the work of a falling weight, of an electric current, and so forth. It was imagined that this conception of energy denoted the discovery of a second quantity fundamentally different from the conception of matter as characterized by the property of mass. But we shall see that still greater unification is, indeed, possible, in that the conception of mass may be referred back to the conception of energy.

These two realms of research, mechanics and electrodynamics, developed along essentially independent lines, until we were one day confronted with the problem of describing events in which motions of bodies occurred in conjunction with electrodynamic phenomena. And then the surprising fact came to light that these realms did not supplement each other in such a way as to allow the observations to be completely and satisfactorily interpreted. The reason for this development may be briefly characterized historically as follows :

The beginnings of mechanics reach back into ancient times. From the beginnings of kinematics, which is a pure description of observed motional events, the science of mechanics, thanks to the researches of Galilei, was developed into a science of dynamics, that is into a study the object of which is to derive the phenomena of motion together with the effects to which they give rise, for example centrifugal forces, from only a few simple principles. Shortly after Galilei had formed the first conceptions that led to the creation of dynamics Newton conclusively set up the essential foundations of mechanics. All that was achieved later constitutes nothing more than an elaboration of his results. So it came about that mechanics was brought to a certain conclusion at the time when electrodynamics indeed only began to bring its data into systematic order. That was about the year 1600. And when Maxwell and Faraday, 250 years later, finally arrived at the point of completing the system of electrodynamics, methods of research had been developed so far that certain maxims could be taken into account logically, which Newton had in mind but which he had not been able to apply.

And thus mechanics, like a beautiful piece of architecture of antiquity, towered above the newer structures of science. But the foundations of this stately building had become so far lost to view that we found no other means of adding the results of new researches than that of constructing a completely new building.

I shall now endeavour to present the foundations of Newtonian mechanics in such a way that the perplexing nature of its relationships come into clear relief. We cannot, of course, in such a critical survey do full justice to its actual achievements.

The first phase in the development of mechanics is marked by the action of Copernicus, who dethroned the world from the favoured position to which it had been assigned in the solar system and, instead, made the sun the central point of the stellar system. The achievement of Kepler, the enunciation of his well-known laws of motion for the planets denotes essentially only a refinement of the Copernican system. Up till that time mechanics had been pure kinematics, that is, it had set out to *describe* motional events, particularly those of celestial bodies, as simply

as possible. It developed into a system of dynamics only after Galilei had set about making the motional events on the earth, too, a subject of study and seeking the laws common to all such motions. He created that fundamental conception which was the condition preliminary to our being able to penetrate further into the nature of motions, namely, the conception of *mass*. Only on the basis of this conception was it possible to understand how the planets moved around the sun although no force was acting other than that which had its centre in the sun. Till that time it had always been imagined that a second force had to be added which acted in the direction of the orbit. This view was first changed when Galilei created the conception of mass, that is, assigned to matter the property which we call inertia. This manifests itself in the circumstance that when matter has somehow attained a certain state of motion it strives to continue in this motion. Galilei hereby created the conception of inertial mass. He created it as an absolute quantity which, as regards its value, is, so to speak, innate in a body and stands in no relationship to the physical conditions of its

surroundings. To study the motion of a body we must know the amount of its mass; whether it is blue or red, solid or fluid, is of no account. In this way Galilei came to formulate his law of inertia which later Newton also made the basis of his mechanics. This law states: "every body that is not subject to external influences persists in virtue of its inertial mass in the state of motion which it has with respect to a co-ordinate system, and it continues to move rectilinearly and uniformly." In view of the state of science at that time this abstraction of Galilei was surprisingly bold. He was the first to recognize quite clearly that the phenomenon of a sphere gradually coming to rest when it has been set running on a horizontal plane is not typical of the nature of every motion. If we could succeed in eliminating fully the friction at the point of contact and the resistance of the air this sphere would continue on its course indefinitely.

But the weakness in the underlying principle of the law of inertia lies just in the circumstance that the abstraction has been carried too far. If a body is to be subject to no external forces at all when the law of inertia is to be

applicable to it then we must exclude not only friction but also the influence of mass-attraction, of gravitation. What guarantee have we that when we release this sphere from the gravitational action of other bodies, so that it is isolated and freely poised in nature, it does not at the same time lose its inertia? Galilei and Newton would presumably have answered this objection by saying: "This is not to be expected; for since, in its motion along the horizontal plane, the sphere is not raised, the phenomenon of inertia has nothing to do with that of gravitation." Thus even if the sphere were completely isolated, we should have every reason for supposing that its inertia would not be affected. But if we enter more fully into the quantitative conditions we find that the data of experience do not speak in favour of such an independence of inertia and gravitation. For if we endeavour to measure the inertial mass of, say, a door, we find that the inertia of an iron gate is different from that of the door of a room and, indeed, that the inertial mass of the iron gate is exactly as much greater as it is heavier. The inertial mass of bodies is strictly proportional to their gravitational mass, or equal

to it, if the unit of measure be suitably chosen. But what is gravitational mass? Newton introduced the term gravitational mass when he discovered that the ground for the fact that "all bodies on the earth fall with the same acceleration" consists in an interaction between the earth and the bodies, an interaction that, indeed, exists between all masses and governs the motion of bodies in stellar space. He set up the law of gravitation, the attractive influence between masses, at a time when the conception of inertial mass had already been moulded, and he formulated this law as follows: two bodies act on each other with a force that is inversely proportional to the square of the distance between them and directly proportional to the product of two quantities that depend on the bodies themselves and that we call their gravitational masses. Every body has a definite gravitational mass "m," and two bodies with the masses m_1 and m_2 exert on each other the gravitational force $\frac{m_1 m_2}{r^2}$, where r is the distance between them. Now, it is characteristic of Newtonian mechanics that this general law of the attraction of masses does not

stand as an equivalent fundamental law alongside of the law of inertia. Thus the motion of bodies under the influence of gravitation does not become the fundamental phenomenon of mechanics but, as before, the place of honour is held by inertial motion, that is, motion that is subject to no forces. The phenomenon of gravitation is only one of the many manifestations of force and is measured by the gravitational mass of bodies. But again, for all these various forces, whether gravitational or elastic, there is a fundamental law which was enunciated by Newton. This third fundamental law asserts that the acceleration which any force imparts to a body is proportional to the force and inversely proportional to the inertial mass of the body. We see that here again the inertial mass appears as the fundamental attribute of matter, whereas gravitational mass appears only when we happen to take into consideration gravitational force from among all possible forces.

But the inertial mass is always equal to the gravitational mass, and it must be borne in mind that if we imagine the never-to-be-observed occurrence of pure inertial motion

to be realized by imagining the sphere to be brought to infinity where it is subject to no forces, the law of inertia demands that this sphere, in virtue of its property of persistence, should move forward uniformly if it happened to have a definite initial velocity. But since the inertial mass is equal to the gravitational mass, and since we should never conceive the idea of assigning gravitation to the sphere (weight) when it is released from the interaction of bodies, it is justifiable to imagine that in this process of complete isolation of the sphere, there is no longer sense in endowing it with inertia. And then the law of inertia would also become meaningless.

The sub-division of motional events into such as are subject to inertia and such as are subject to gravitation led to the consequence that the fact of the equality of inertial and gravitational mass which Newton had discovered and proved with the greatest certainty was not further included at all in the fundamental laws of mechanics. This equality of inertial and gravitational mass occurs, in a certain sense, only as an accident of Nature. In itself this circumstance could be regarded merely as an error of style in Newtonian mechanics.

The following doubtful point, however, arises : since Newton did not succeed in combining inertial mass and gravitational mass—which are always equal to each other—to a unified fundamental conception he also did not succeed in banishing the conception of absolute motion out of mechanics. What we observe, however, is always only the relative motion of bodies to each other. Whether we are dealing with translations or rotations we always perceive only the motions of bodies with respect to each other, and we should expect that the laws of mechanics themselves are only assertions about the motions of bodies with respect to each other. For in natural science we strive to bring into relationship only that which is possible of observation. But Newtonian mechanics could do justice to such a view of motional events only in so far as uniform translations were involved. If, for example, a man is seated in a railway carriage he cannot test whether the train is in motion by observing whether a train is standing at rest on a neighbouring line. He has experienced only too often that it is the latter train that has started to move and not his own, without

being able to decide otherwise than by fixing his eye on the platform. In this the fact of the relativity of uniform motions is clearly manifested. Children, who are more directly subject to sense-impressions, are given to the continual illusion that the surroundings and the trees are rushing past the train and not that, conversely, the train is passing over the ground. If the laws of Newtonian mechanics did not give expression to this relativity of uniform translations it would contradict one of the simplest and earliest facts of experience. As a matter of fact Newtonian mechanics *does* take it into account; how it does so, we shall see in the sequel.

But why does it not enable us to regard rotations—which, as we know, are to be reckoned among accelerated motions—as relative motions? Newton came to the conclusion that it was not possible, and he argued this on the grounds of the following experiment: he set a cylindrical vessel which was filled with water and suspended by a thread into rapid rotation. At first the liquid took no part in the rotation because the friction at the walls of the vessel was not great. So long as this condition existed the surface of the water

remained completely plane. Only when the water took part in the rotation did the centrifugal forces in the water begin to manifest themselves; the liquid rose at the walls and its free surface formed a paraboloid of revolution. He concluded from this that the rotation of the vessel relative to the water is not equivalent to the rotation of the water relative to the surrounding space. For centrifugal forces present themselves only in the second case. Thus what distinguishes rotations from translations is that the relativity of the translations is already given by the kinematic view alone. In the case of rotations, however, certain forces appear in addition, and according to the views of Newton their occurrence is the reason why we are not in a position to regard the rotations, too, as relative motions. So it suggests itself to us to ask:

(1) What is the characteristic feature of these forces that occur in rotations?

(2) Does the occurrence of centrifugal forces actually make it impossible for us to regard rotations as relative motions?

According to the view of Newtonian mechanics centrifugal forces are a consequence

of the inertia of matter. On account of its inertia each particle of the earth, for example, strives at every moment to move on rectilinearly, that is, tangentially to the earth's surface. But its relationship with the other particles compels it to move in a circle. This requires a force that is proportional to the inertia, and this force presents itself as the centrifugal force. Thus the centrifugal force is a case in which the inertia of matter manifests itself in a pure form. If the earth were poised in space at an infinite distance from all other masses we should nevertheless be able to perceive the rotation of the earth because we could prove the presence of centrifugal forces at its surface. Hence we must assume that there is some relation between matter and space such that if the bodies execute this definite kind of motion relatively to the absolute space, centrifugal forces are called up. Consequently we are compelled to talk of absolute motions as distinct from relative motions. But we cannot form the vaguest idea of this conception of absolute motion in space. What we can observe are exclusively motions of bodies with respect to each other.

Yet, if we attack the root of the question and ask whether Newton's method of deduction cannot actually be circumvented, we find that it is just the equality of inertial and gravitational mass, discovered by Newton himself, that gives us the key of the problem as to how rotations are to be regarded as relative motions. For what we really observe here on the earth is the weight of bodies, conditioned by the attraction of the earth, and superposed on this there is yet a second force that does not act towards the earth's centre, but in the direction of the parallels of latitude; they are called up by the rotation of the earth about its axis. Since, however, the inertial and the gravitational mass of matter are exactly equal, we have no means at all of making a difference between gravitational force and this centrifugal force due to inertia. Indeed, we measure only the sum of these two forces and cannot separate them from each other. It would therefore be just as admissible to interpret the observed facts as follows. On the earth we observe on the one hand a gravitational force directed at the earth's centre, on the other hand, however, a second gravitational force acting in the

direction of the parallels of latitude. We should then have to consider how this second force actually comes about. Now, it would be quite justifiable to say that this second gravitational force, which we hitherto called centrifugal force, is called up by the masses of the universe circulating round the earth. On account of the equality of the inertial and gravitational mass the following two views are possible:

(1) The earth rotates about its polar axis once in twenty-four hours; owing to the inertia of its matter centrifugal forces manifest themselves on its surface. Or,

(2) all bodies rotate about the earth once in twenty-four hours and, in consequence of the general gravitation, they exert definite forces on the earth. These gravitational forces take place in the same way as the so-called centrifugal forces in the first view. This standpoint certainly requires a complete fusion of the two fundamental conceptions, inertia and gravitation.

Thus we see even in our brief discussion how a long chain of difficulties follows in the train of the law of inertia, so that we must admit that these foundations are really quite unintelligible.

A further doubt arises in addition to these. The law of gravitation that we formulated at the outset asserts nothing about how the force of gravitation is actually transmitted through space. A force, indeed, whose behaviour is regulated by this law must propagate itself with infinite velocity through space. If the sun were suddenly to burst into several pieces, then the altered attraction would at once, without any loss of time whatsoever, penetrate to the furthermost corners of the earth, but such a law contradicts the postulates that we must impose on all physical laws. We expect that all effects that we observe have a finite, even if extremely great velocity of propagation. Only this allows us to determine completely the physical state at any point by means of that of its immediate neighbourhood. Even if we often formulate laws so that this postulate does not appear explicitly—we need only think of the law of moments of a lever, which states that the action is proportional to the force and the finite distance from the point of support—yet we know quite well that the essential point involved is that the connection through the arm of a lever is rigid, and that

consequently the state of tension is transmitted by it. If this arm of the lever were not present, and if there were no material connection at all between the point of application of the force and the point of support, the law of moments for levers would appear to us meaningless. Newton's law of gravitation is in truth nothing more than a law of levers in which there is no connection between the law of support and the point of application of the force.

When we criticize the foundations of Newtonian mechanics in this rigorous fashion, it is difficult for us to understand how its achievements could have been so great. It seems incomprehensible that just that discipline that has been built up exclusively on the hypotheses of Newtonian mechanics, namely celestial mechanics, succeeded in acquiring a distinctive position among all branches of physical science in virtue of the accuracy with which it describes the motion of the heavenly bodies. If we consider, for example, that in the course of a century Mars cannot move further even to the extent of $\frac{1}{100000}$ of its orbit than theory demands, without the astronomer noticing it, we may

gauge what degree of accuracy is entailed in Newton's law.

About seventy years ago, when the existence of a hitherto unknown planet had been successfully predicted from observations of disturbances in the orbital motion of Saturn, and when, a few days later, it was actually discovered at the point which theory had determined beforehand, confidence in the theory became so great that it was believed that every disagreement between observation and calculation was necessarily due to yet unknown masses. But one disagreement manifested itself which, in spite of all efforts, could not be accounted for in this way.

The orbit of Mercury would have to be an ellipse at rest relatively to the stellar system, if the sun alone were acting on it. In consequence of the attraction of the remaining planets, however, this Kepler ellipse is set into motion and slowly rotates, namely, to the extent of a few hundred seconds of arc per century. But observations show that this rotation of the major axis of Mercury's orbit is greater than it should be by about forty-three seconds of arc per century. Inasmuch as the existence of Neptune had been pre-

dicted shortly before owing to a similar disagreement between observation and calculation, it was originally believed that it would without doubt be possible to explain the anomaly of the motion of Mercury by the action of yet unknown masses. But all efforts to discover these masses have been futile up to the present day. We have gradually gained the conviction that such a mass does not exist. For if it were concentrated in a planet it would have to be so great that we could not have failed to observe it during the many solar eclipses.

Nevertheless no pressing need was felt to proceed to a revision of Newtonian mechanics on account of this discrepancy between calculation and observation. It was so enormously successful in other respects that in the course of time almost everyone forgot to criticize its foundations and thus to prove the necessity of such a revision. Electrodynamics gave the first stimulus to this step. This stimulus came from a direction from which Newtonian mechanics had led us to expect it least.

As already mentioned, Newtonian mechanics satisfies the postulate that all motions are to be regarded as relative motions of bodies

with respect to each other only in so far as uniform and rectilinear motions come into question. It thus entails a principle of relativity for this class of motions. It was just in this point, in which mechanics satisfied the fundamental postulate, that the accuracy of its laws began to be looked on with doubt. We must, therefore, enter more closely into the meaning of this principle of relativity. As remarked above, observations compel us to regard translations as relative motions. This brings about the common result that observation allows us to determine motional events on the earth, but not its own motion in space. If this motion of the earth were to make itself noticeable in all events that take place on it we should probably not yet be in a position to master the simplest process of motion completely. For not only does the motion of the earth come into question, but also the motion of the sun, of the stellar system, and finally even the motion of the whole stellar system, which is at present still quite unknown. But the principle of relativity asserts: whether we regard the earth as at rest or in motion is a matter of complete indifference for the description of motional

events on the earth. That is, the laws which govern these motions retain their form if we transform co-ordinate relations, namely, the relations that fix the point of happening and the course of the event, from the case of the earth considered at rest to that of the earth assumed in motion. Hence the principle of relativity resolves into two parts:

(1) Into the *postulate* of relativity which determines for what kind of motion systems are to be regarded as equivalent from the point of view of mechanics. Newton's mechanics limits its postulate of relativity to systems that are moving uniformly and rectilinearly with respect to each other.

(2) Into the system of *transformation formulæ*, according to which the transformation of co-ordinates is to be undertaken for equivalent systems.

When Galilei and Newton formulated their principle of relativity it was believed that the setting up of these transformation formulæ was a problem of pure mathematics. If x, y, z are the co-ordinates of any point of space, referred to a rectilinear system of co-ordinates, whereas x', y', z', are the co-ordinates of the same point referred to a

second system that moves, relatively to the first, parallel to the x-axis with the constant velocity v, and if the co-ordinate axes are parallel in these systems, then the co-ordinates of either system are to be transformed into those of the other by means of the formulæ :

$$x' = x - vt, \qquad y' = y, \qquad z' = z;$$

to these there is to be added a further formula that was not specially mentioned earlier, namely, $t' = t$. According to these formulæ, for example, the velocity q' with which any event in the second system transmits itself in the direction of the x-axis is $q' = q - v$, that is, always different from q, the velocity of transmission in the initial system, if both systems are moving with respect to each other. We shall see that it is just in this point that the change of view becomes necessary and that, in a certain sense, the battlements of classical mechanics were first shattered. But we shall leave this problem for the next lecture.

To-day we have yet briefly to consider the development of electrodynamics parallel to mechanics up to the time when the collision occurred between them which gave rise to Einstein's theory of relativity.

As already stated, electrodynamics began to be treated systematically only after Newtonian mechanics had, as it were, already been brought to completion. But the phenomena with which electrical theory was confronted were so abundant and so varied that this theory did not run the risk of becoming a structure involving too formal conceptions, but was compelled to work its way through the dense forest of manifold single phenomena by very laborious individual efforts.

The observations from which electrodynamics arose were not essentially very different in truth from those on which celestial mechanics is founded. It was observed that in certain cases substances either attracted or repelled each other, and this was traced back to definite interactions by creating the conception of electric charges. But one did not fall into the temptation of defining this peculiarity of substances as an absolute property of matter, that is, as one independent of other physical conditions. For it was soon discovered that these charges could be given to or taken away from substances at will. Consequently it was first necessary to work through all the various peculiarities of the

different substances before a systematic treatment of these phenomena could be effected. The law which, it is true, was used at the very beginning to calculate the forces of attraction between two charged small spheres was no other than Newton's Law of Gravitation, in which the conception of gravitational mass had been replaced by that of the electric charge. But even Faraday realized that the amounts of these forces depended, indeed, essentially also on the medium that occupied the space between these two spheres, and he thus prepared the way for the view that electromagnetic influences are transmitted continuously through space.

The phenomena of electrodynamics, too, exhibit the remarkable duality of two ultimately different forms, namely electric and magnetic phenomena, just as in mechanics inertial and gravitational phenomena are to be distinguished. But whereas in mechanics they appear together, without intelligible connection, the fundamental problem of electrodynamics has been from the very outset to determine the correlation between electricity and magnetism, and to find the fundamental laws that give the numerical connection

between these two forms of phenomena. And although formulæ were here used that were derived from the view of action at a distance, yet one was always conscious that these formulæ were purely formulæ of calculation and did not represent the final form of these laws. The foundations of electrodynamics were regarded as sufficiently established only after Maxwell had formulated its fundamental laws in terms of contiguous action, i.e. action by contact. Maxwell's four laws state the following:

(1) *In the neighbourhood of an electric charge electric fields are generated. The strength of these electric fields is conditioned among other things by the surrounding matter.*

(2) *There is no free magnetism.*

(3) *An electric current calls up a magnetic field around itself.*

(4) *A magnetic displacement-current generates an electric field.*

Thus electrodynamics reached completion on the foundations of the view of contiguous action, which could not come into its own in Newtonian mechanics. It had then to be decided which of these two regions better satisfied the conditions that natural science imposes on a physical theory.

A comparison of these two theories became feasible when Maxwell's theory had made it possible to regard all optical phenomena as manifestations of electromagnetic processes of radiation. It thereby raised the question as to how electrodynamic processes of radiation take place between bodies that are themselves moving. And mechanics had to furnish the material for describing such events, in which mechanical and electrodynamic events occur in conjunction with each other. The first to attempt to set up a theory of such phenomena was Heinrich Hertz. But his theory was not in agreement with observation because he too boldly sought to set up at once a theory which allowed the bodies to have any motions whatsoever. When H. A. Lorentz later proposed a theory new experiments had been carried out successfully and new phenomena had been discovered which enormously extended our range of survey of such events.

In particular, the discovery of the atomistic structure of electric charges had been made, and these electric atoms had been observed in motion, and, indeed, velocities had occurred which were incomparably greater than those

which mechanics had hitherto had to consider in its laws. It then became apparent that all attempts to describe these events satisfactorily by means of mechanics were doomed to failure. Many years of work were spent in efforts to overcome these difficulties. Faith in the laws of mechanics was so great on account of their success in other directions that it was only with great reluctance that the view was accepted that the difficulties lay concealed in the foundations of Newtonian mechanics. It was due to Einstein's researches that the source of the difficulties was first discovered to be a gap of a fundamental kind in Newtonian mechanics, and Einstein thus broke the spell which had hitherto hindered the remodelling of the Newtonian doctrine. In the next lecture we shall deal with the first stage in the revision of the foundations of Newtonian mechanics. It is characterized as the revision of the Newtonian principle of relativity so far as uniform rectilinear motions are concerned. In the third lecture we shall enter into the general theory of relativity which alone has made it possible to regard all motions, and hence also rotations, as relative motions of bodies with respect to each other.

SECOND LECTURE

THE SPECIAL THEORY OF RELATIVITY

THE first lecture was essentially devoted to general considerations for it seemed necessary to prepare the ground from which the new views of the theory of relativity have emerged. We saw that two complexes of phenomena are comprised in the physical happening of the world, the phenomena of mechanics and those of electrodynamics. Both branches developed fully independently of each other. Mechanics was constructed entirely on the view of action at a distance, and it was characterized by the circumstance that two fundamental conceptions occurred in it which, according to observation, had necessarily an intimate connection, namely, the inertial and gravitational mass of bodies, yet it did not succeed in com-

bining these two conceptions into a single idea.

Electrodynamics, on the other hand, was based exclusively on the conception of contiguous action, and its fundamental laws contained definite assertions about the relationship between electricity and magnetism, which stand opposed to each other in electrodynamics in much the same way as inertia and gravitation in mechanics.

When we were then confronted with the necessity of describing events in which mechanical and electrodynamic processes occurred in conjunction, it became apparent that it was not possible to combine these two regions to form a uniform world-picture.

This difficulty was removed by Einstein. We shall endeavour to follow the course of development so that the relationship with experiment and observation is made directly evident.

The most important discovery of Maxwell's theory was that the propagation of light may be regarded as a process of electromagnetic radiation. It is in just this region that we encounter phenomena in which electromagnetic events occur in conjunction with

motional events, as, for example, in the radiation of the sun and the stars. Essentially three questions were thereby raised.

(1) Does the velocity of light depend on the motion of the source of the light ?

(2) Does the carrier which transmits the light in bodies participate in the motion of the matter ?

(3) How does the influence of the earth's motion affect the propagation of light on its surface.

Question 1.—Only fairly recently it was possible to hold the view that the motion of the source imposed itself directly on the transmission of the light. Thus it was believed that—as in the case of a rifle-bullet—the motion of the source possibly added itself to the velocity. But a few years ago a criterion was discovered which may practically be regarded as establishing that the propagation of light is in no way influenced by the motion of the source. Let us consider a double star in the heavens, the bright component of which has an orbital component of 50 kms. per second, and let us assume that the time of revolution of the system is two days. These are conditions such as

we often observe. We next assume that the velocity of the source superposes itself on that of light. The following will then occur. In the constellation in which the star moves away from us with the greatest velocity, that is, 50 kms. per second, it emits light which would rush towards the earth with a velocity of (300,000 − 50) kms. per second. A day later it is situated on the opposite side of the central body and moves towards us with a velocity of 50 kms. per second. In conformity with our assumption the light-rays from this constellation traverse space with the velocity of (300,000 + 50) kms. per second in the direction of the earth. The result will be that, although the rays from the first constellation left a day earlier than those of the second, the latter, having a greater velocity, will reach the earth earlier if they have enough time to catch up and pass the rays of the first. This would happen in 3000 days, that is in eight years. Thus if the earth chanced to be at a distance of eight light-years from the star, then the rays of light would arrive simultaneously, and the inhabitants of the earth would see the same star appearing on the right and left simul-

taneously. The fact that such an anomaly in the motions of the system of a double star has never been observed, although most stars are much more distant still, is convincing evidence that the light which reaches us from the stars is totally unaffected by their conditions of motion. It has been estimated to what extent the velocity of light would be influenced if the velocity of the source were only partly superposed on that of light, and nowadays we have the certainty that this fraction cannot amount to more than one millionth of the velocity of the source. Thus the first question is to be answered in the negative.

QUESTION 2.—It is much more difficult to answer this question ; nor is it so simple to understand the result. On the basis of Newtonian mechanics there are two possibilities : the carrier in the transmission of the light—for example, in moving water—participates in the motion of the water *or* it remains at rest. Now, the following simple experiment was carried out by Arago :

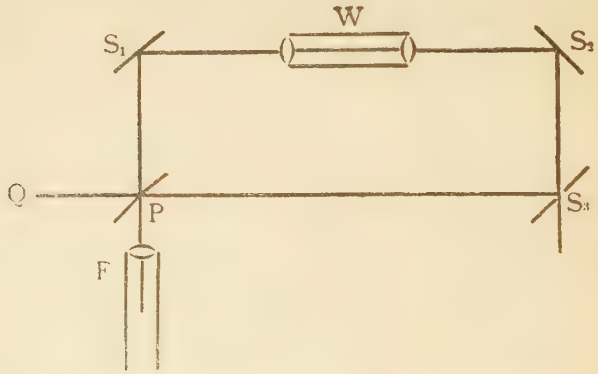
Let us assume that the ether is at rest. An astronomer directs his telescope at a star which happens to be placed so that his line of vision is in the direction of the momentary

motion of the earth round the sun. The relative motion of the telescope to the star amounts to 300,000 kms. *plus* the velocity of the earth. Six months later it will be 300,000 kms. *minus* the earth's velocity. Now, since the refraction of the light depends on the velocity of the light in the lens of the telescope, the point of focus would have to be different at these two points of time, and the astronomer would be compelled to focus the star afresh after every six months. Nothing of this sort has ever been observed. Thus the ether is not completely at rest. Then only the second possibility remains, namely, that the ether is carried along by the moving matter. To determine the extent of this convection various experiments have been carried out, as, for example, those by Fizeau and Hoek.*

Hoek made the following experiment. Light from a source Q falls on a plate P inclined at an angle of 45°, which allows a certain fraction to pass through and reflects a part. Thus

*A detailed account of these experiments is to be found in "Die Relativitäts-Theorie Einsteins" by M. Born (Springer, Berlin) which will shortly appear in English (published by Messrs. Methuen & Co. Ltd.).

some of the light is forced to travel along the path $PS_1S_2S_3P$ whereas the other part traverses the same route in the direction $PS_3S_2S_1P$. Both rays are made to pass through a tube W containing water in their passage. The apparatus is placed so that the direction of the tube coincides with the earth's motion. The rays are then again combined



and the interference fringes that occur are observed in the telescope F. We now consider the three cases that are possible :

(1) *The ether is completely at rest.* If this is the case then the velocity of light in the water which participates in the motion of the earth is $c_1 = \frac{c}{n}$, where n denotes the coefficient

of refraction of the water. If the apparatus is placed so that the light traverses the distances PS_3 and S_2S_1 in the direction of the earth's motion, then the velocity of light relative to the earth is equal to $c \pm v$ in its course through the water, according as the ray is moving in or opposite to the earth's direction of motion.

(2) *The water takes the carrier of light completely along with it.* Then the velocity of light relative to the earth is equal to c_1 in the water, but $c_1 \pm v$ relative to the outside ether.

(3) Let us consider the more general case in which matter carries the ether along with it, but not to the full extent of its own motion. Let the velocity of light in the water, which, of course, is moving with the earth, be $c_1 + \phi$, where ϕ measures the extent of the convection of the ether in water. Then the velocity of light relative to the earth is $c_1 + \phi - v$ for a ray which is transmitted in the direction of the earth's motion. We have the case of complete rest when $\phi = o$, whereas when $\phi = v$, we have the second case of complete convection. The experiment has to decide which value ϕ has actually. What we compare are the times taken by the

light to traverse the same distance $PS_1S_2S_3P$, once in the direction as written, and once in the opposite direction. We ascertain the following :—

If l is the length of the ray's path in the water, then the time required by the ray for this part of its journey, occupied by water, is :

$$t_1 = \frac{l}{c_1 + \phi - v}.$$

On the return journey S_3P this ray does not traverse the water at all, but only the air ; consequently the corresponding time on the other side is

$$t_2 = \frac{l}{c + v}.$$

The total time taken by this ray is

$$T_1 = \frac{l}{c_1 + \phi - v} + \frac{l}{c + v}.$$

The total (and this is all that we are concerned with) time taken by a ray to traverse the same path $PS_3S_2S_1P$ in the reverse direction is

$$T_2 = \frac{l}{c - v} + \frac{l}{c_1 - \phi + v}.$$

The apparatus was adjusted correspondingly, and was turned through 180°, so that

its orientation towards the earth's motion became changed. If T_1 and T_2 were different then the interference fringes would have to shift during the turning of the apparatus. Yet no shift manifested itself. It was found that $T_1 = T_2$.

If we calculate ϕ from this equation we get

$$\phi = \left(1 - \frac{1}{n^2}\right) \cdot v,$$

that is, in moving matter ether is not completely at rest. Arago's experiment had shown the same result. The ether is not, however, completely carried along, but only to the small extent $\left(1 - \frac{1}{n^2}\right)v$, which depends on the coefficient of refraction of the substance that is traversed by the light.

Hence question 2 on p. 34 may be answered in the negative, in so far as we fix our attention only on the most easily pictured case of the complete convection of the ether. But the experiment also teaches us that this method offers us no means of determining the earth's motion by observing optical phenomena on the earth. The process of the propagation of light allows us to determine the motion

of the earth in space just as little as do observations of motions on the earth. The observed event takes place quite independently of the orientation of the apparatus relative to the direction of the earth's motion. This leads on immediately to :

QUESTION 3.—How does the motion of the earth influence the velocity of light on its surface ? Can we not, after all, by arranging some apparatus appropriately, prove the motion of the earth from the behaviour of the propagation of light on its surface ? The decisive experiment was made, as we know, by Michelson, and later repeated by him in conjunction with Morley. We shall illustrate the nature of this experiment by the following instructive example.

We suppose that on an island in a wide current that is flowing with a velocity of 3 kms. per second there are two oarsmen who can row with exactly equal speed. They agree to row to two buoys, the one rowing up-current, the other perpendicularly to the current, the distance of each buoy being 4 kms. from the island. They start simultaneously. The first, rowing perpendicularly to the current, will steer at a definite angle to

the current from the very beginning to avoid being taken out of his course. Let us assume that he reaches the buoy in exactly one hour. He will then have traversed 5 kms. in one hour, for since the buoy is distant 4 kms., and the stream flows 3 kms. in one hour, the actual distance traversed is (by Pythagoras' theorem) $\sqrt{3^2 + 4^2} = 5$. Thus the oarsman will have reached the buoy in one hour and will have returned in another hour. Now, the second oarsman rows with the same velocity of 5 kms. per hour against the current. After having rowed two hours he has traversed 10 kms., but in this time the stream has carried him down-stream 6 kms. The buoy, which is distant 4 kms. will thus be reached by him only after two hours. However quickly he may row back he can never reach his starting-point simultaneously with the first oarsman. In general this argument means that when an event happens with a definite velocity of its own against the flow of the ether in which the earth is swimming, then that ray which is directed against the current (the "ether-wind") will require longer to pass to and fro over a definite distance than the ray which moves

perpendicularly to the current over the same distance. In the arrangement of Michelson and Morley two rays of light, just like the two oarsmen, were made to traverse the same distance, in one case in the direction of the earth's motion, and in the other perpendicularly to the same distance. It was to be expected that the one ray would require longer than the other. But no difference manifested itself. Thus the above argument does not apply to light. Light does not trouble about the current of ether that it encounters. The times taken by the light are quite independent of whether the light is being propagated parallel or perpendicularly to the earth's motion.

This result, again, may be interpreted in two ways. We observe that the velocity of light behaves on the earth like a universal constant in nature that in no wise troubles about the phenomena of motion. And then this experiment teaches us that we are not in a position to prove, by observing optical phenomena on the earth, the motion of the earth. Thus here, too, we have the same result as we get from the ordinary motional events on the earth.

Let us recapitulate what the experiments on the propagation of light on the earth teach us. The results are :

(1) The velocity of light is completely independent of the motion of the source.

(2) The velocity of light on the earth is completely independent of the state of motion of the earth ; it is the same in all directions.

(3) The velocity of light in moving matter depends on the velocity of matter in a rather complicated way. The carrier in the transmission of light is neither swept along completely by the matter nor does it remain fully at rest.

This state of affairs presents us with a new question.

Is a velocity that manifests the properties of light as characterized in the above way possible at all within the realm of Newtonian mechanics ? Definite formulæ of Newton's system give us an unmistakable answer to this question. This answer is : " no." For if the velocity of propagation of an event is q in one system then in another, which is moving relatively to the first with a velocity v, the velocity of propagation is $q' = q \pm v$, that is, always different from q, if the velocity v of

both systems with respect to each other is greater than zero.

Let us suppose that we are in a moving railway carriage and that we roll a ball along the floor. The relative velocity of the ball with respect to the floor is a certain number of meters per second, but is always different with respect to the embankment so long as the carriage is in motion. Only when it is at rest is the velocity of the ball the same relatively to both the floor and the ground. But light behaves quite differently. Here we have gained an experience that does not allow itself to be brought into agreement with our hitherto accepted formulæ, and there is, of course, no question as to what our decision must be. We have to arrange our system of physics so that it does justice to the facts that have been established with certainty. And if we arrive at the conclusion that our formulæ do not do justice to our new experience or observation, then our physics must adapt itself to this new fact of experience. Consequently the further question arises: what are the physical assumptions that led to the formulæ that have hitherto been used in Newtonian mechanics, and which of these

assumptions cannot be maintained? Up till recently it had, indeed, been believed that the formulæ of Newtonian mechanics which enabled us to transform the space- and time-measurements from the moving train to the embankment at rest, or, generally, from one system of reference to another, had been obtained by pure mathematical analysis, and were free from hypotheses. That this is not so may easily be seen from the following reflection.

A mathematician goes to a physicist and asks for a problem in connection with the questions here treated, and the physicist propounds the following question. What are the most general equations of transformation which allow us to transform the space- and time-measurements from one system S_1 to a second system S_2 which is moving relatively to the first uniformly and rectilinearly? To make quite sure that the solution sought may be applicable to the events that occur in nature, definite conventions must be made. Thus it is assumed (1) that the form of the required formulæ is independent of the direction in which the two systems S_1 and S_2 are moving with respect to each other. That is,

space is to be isotropic and is not to have properties such as we observe in the case of crystals. Hence the state of motion of both systems can receive expression in the formulæ only by means of the amount of their relative motion. We assume (2) that the formulæ are to have the property that when they are used to carry out calculations from one system S_1 to a second S_2 and then to a third system S_3, this repeated application of the same transformation formulæ is to give the same result as if we had passed straight from the system S_1 to the system S_3. We say that the transformation formulæ are to have the *group property*.

A few days later the mathematician again appears and says that these two conventions do not suffice to solve the problem uniquely. For it must first be decided whether there is a finite velocity which remains constant in value, quite independently of the choice of the system. For if such a velocity exists it would play a very particular part.

The physicist would answer that if the form of the transformation formulæ really depends on whether a finite velocity retains its value and is not influenced by the action

of the transformation formulæ or whether this is so only for the exceptional case of an infinitely great velocity, then the decision on this point can come only from experience. For it alone can teach whether or not there is in nature a finite velocity that behaves like a constant with regard to the transformation formulæ. And he will find that actually the velocity of 300,000 kms. per second, with which light propagates itself, plays this particular part in nature. The mathematician will then give him as the transformation formulæ for passing from the space- and time-measurements of one system S_1 to a system S_2 moving relatively to the first, those that were set up by Lorentz.

Thus on the grounds of perfectly general reflections, and by keeping strictly to experience we arrive at the conclusion that the transformation formulæ with which we have to calculate in nature are those of Lorentz and Einstein, and not those of Galilei and Newton.

The Lorentz-Einstein transformation formulæ are, for the special case when the two systems S_1 and S_2 are moving parallel to their x-axes with the constant relative velocity v:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}},\ y' = y,\ z' = z,\ t' = \frac{t - \frac{v}{c^2} \cdot x}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

The corresponding transformation-formulæ according to Galilei and Newton are: $x' = x - vt$, $y' = y$, $z' = z$, $t' = t$. Here x, y, z, t and x', y', z', t' denote the co-ordinates of the space-time measures in the two systems S_1 and S_2, respectively, and c denotes the velocity of light *in vacuo*. As we see, the transformation formulæ of Lorentz and Einstein pass over into those of Galilei and Newton if we allow the quantity c to become infinitely great in the former. This corresponds to the hypothesis contained in the whole of Newtonian mechanics, that no finite velocity may play the part of a universal constant in nature.

Now, Newtonian mechanics had given expression to the empirical fact of the relativity of uniform and rectilinear motions in a principle of relativity that required the invariance of its equations of motion with respect to the transformation formulæ of Galilei and Newton. But these transformation

formulæ must be given up. The empirical fact of the relativity of uniform rectilinear motions, of course, remains unimpeached. It must receive expression in the laws of motion of mechanics in that the latter must preserve their form with respect to those transformation formulæ that effect the transition from one system to another which is moving relatively to it. Thus a form must be found for these laws of motion such that it remains preserved when the Lorentz-Einstein transformation formulæ are applied. On the other hand, it had long been known that Maxwell's equations of electrodynamics fulfil the condition of invariance with respect to such transformations. But it was not at that time known what was the meaning of these Lorentz transformations. It was Einstein who first realized that they are indeed the formulæ according to which the co-ordinates of systems moving uniformly and rectilinearly with respect to each other are to be referred from one to the other. He realized this after experiment had taught us that the velocity of light may be regarded as a universal constant. Since a principle of relativity actually seems to exist in nature, the formulæ

of mechanics must be modified so that they preserve their form after the new transformation-formulæ have been applied.

In this way we arrive at the so-called special principle of relativity which asserts that *rectilinear uniform motions are relative motions*. The formulæ of physics must remain unaltered if we transform the space-time co-ordinates in them by means of those formulæ which refer equivalent systems to one another; and these are the Lorentz-Einstein formulæ.

In itself this step is of no particular importance, but results of incredible consequence follow from the new transformation formulæ. As we know, the greatest attention was excited by the following. Let us assume that, with the help of the new formulæ, we wish to calculate the length of an object, once from the system at rest, and secondly, from the moving system. We set a measuring rod along the x-axis in a rectangular system of co-ordinates, so that its zero point coincides with the origin of the system. If its length is 1, then the other end of the rod will be at the same point $x = 1$ in the system S in which we suppose the rod to be at rest. What point does this

other end occupy in a second system S′ that is in motion relatively to the first? For the sake of simplicity we shall count our time from the moment when the origins of both systems coincide, and we shall assume that the motion occurs with a constant velocity "v" parallel to the x-axes of both systems. According to the formulæ of Newton we should have: $x' = x - vt$, $t' = t$. At the moment $t = 0$, $t' = 0$, we have $x' = x = 0$ for the zero-point of the rod, and $x' = x = 1$ for the end-point, that is, the length of an object is completely independent of whether we measure it from the system which is at rest or from that which is in motion. But what do the new formulæ give us? According to them, we get

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

If we again count the time from the moment when the origins of both systems coincide, then we get for $t' = 0$ that $t = \frac{v}{c^2}x$. If we insert

this in the first equation, then quite a simple calculation gives us :

$$x' = x\sqrt{1 - \frac{v^2}{c^2}} = 1 \cdot \sqrt{1 - \frac{v^2}{c^2}},$$

that is, the length of the rod in the moving system is shortened compared with that in the system at rest, the factor of shortening being $\sqrt{1 - \frac{v^2}{c^2}}$. Exactly the same result follows if we apply our argument to a time-length instead of a space-length. We get for the time "t" of happening of an event in the moving system as compared with that in the system at rest : $t' = t\sqrt{1 - \frac{v^2}{c^2}}$, that is, the duration of an event, too, is dependent on the state of motion of the observer. This result excited great attention because it had hitherto been believed that the length or the volume of a body were quantities that did not depend on the type of observation at all, but had an absolute meaning, just as had been assumed for the inertial mass in Newtonian mechanics.

The new formulæ of Einstein's special theory of relativity teach us that this is not

the case. We shall illustrate this new discovery by means of a quite simple example. An astronomer observes the appearance of a sun-spot. Suppose that he observes it at eight minutes past twelve. Since he knows that light takes about eight minutes to travel from the sun to the earth, he will affirm that the moment that the sun-spot begins to appear is to be reckoned back to 12 o'clock. An absolute meaning was assigned to this fixing of the moment, that is, it was believed that every observer would arrive, independently of his state of observation and motion, at the same moment for the appearance of the sun-spot, it being assumed that all clocks were in unison and went at the same rate for all observers. But the following second case will demonstrate that this deduction is false, and that we have no authority for drawing conclusions of this absolute type in physics.

We assume that two physicists have agreed to observe a storm that occurs at just the moment when the one physicist is sitting in a railway carriage and passes the house of the second physicist who is at rest. Further, we assume that at the precise moment when the carriage containing the travelling physicist

is passing the point at which the resting physicist is situated on the embankment (in his house), two flashes of lightning strike the embankment, one to the right and one to the left of the traveller. Suppose the strokes are such that the resting physicist observes them to have struck the embankment absolutely simultaneously. If he then measures his distance from the two points at which the lightning has struck he will find that his point of rest in the house is exactly mid-way between them. If the distance between the two points is L metres, and if his watch indicates 12 o'clock at the moment at which the strokes occur then he will say: the lightning strokes occurred at 12 o'clock *minus* L/c seconds. At the same moment the carriage containing the other physicist is to have passed directly in front of the resting physicist. What will the moving physicist observe? His train is moving towards the one stroke and away from the other. Hence the former flash will be seen by him sooner than the latter, and he will conclude that the former occurred earlier than, and not simultaneously with, the latter. We are here dealing with the most primitive fact of the relativity of

simultaneity. As human beings we are compelled to build up our whole physics on experiences and observations which, owing to the propagation of light, we are able to gain from physical nature, and hence it is not possible for us to form a conception of time that is independent of the peculiarities of optical transmission. There is no sense in saying : if there had been some means by which the moving physicist had been able to observe the lightning without having to use light for transmitting the signals, then he would have been able to establish that both strokes actually occurred simultaneously. This possibility never arises and therefore events that are simultaneous for the physicist at rest are never so for the moving physicist. Thus *all* events will take a course for a moving physicist somewhat different from that for a fixed physicist, and the moving observer will arrive at a somewhat different description of events. Initially, opposition was raised to this necessary conclusion, and it was thought possible to define the conceptions of duration and of the length of a body in such a way that the relativity of these conceptions would not

become appar nt. We could never enter the realm of physics with such conceptions which, as a matter of principle, we could, of course, form. For what we measure as the length of the rod when it is at rest, and what we establish as its length when it is moving past us are two different quantities, which always come out differently numerically if we make use of the means offered to us by nature. Thus, physical conceptions formed " absolutely " in this way would have nothing in common with the properties of material things as subject to observation. The length of a body, the duration of an event are not assertions about absolute attributes of things, but assertions about relations that exist between the observer and the thing observed. The relations depend very essentially on the state of motion of the observer and the properties entailed in the velocity of the signals used by him.

Perhaps, if only such general discoveries had been the deciding features of the special theory of relativity of Einstein, a great number of scientists would have said : In principle we admit that the new view is justifiable, yet this theory of relativity is only of small

practical value to us. For in nature we have always to deal with velocities that are so small compared with that of light that we may always neglect the members $\frac{v^2}{c^2}$ and $\frac{v}{c^2}$, and then, as we see, the Lorentz-Einstein formulæ pass over into those of Galilei and Newton. We may also formulate this thus: the difference between the earlier formulæ and those of Lorentz and Einstein consists in the circumstance that the older mechanics regarded the velocity of light, in a certain sense, as infinite. In practice this assumption will probably almost always be admissible, but in principle it represents a hypothesis that may not be made.

The new theory of relativity gains its great physical significance from a discovery of enormous consequence. For it is a direct consequence of its formulæ that the quantity which we call energy, and according to which we measure amounts of work in physics has the property of inertia in common with matter. Whereas it was hitherto believed that matter and energy were two fundamentally different physical quantities, the special theory of relativity prepares us for the view that they

are not different by nature. And the time is not distant when we shall presumably be in a position to do without one of these conceptions, that is, when we shall be able to regard energy as a form of appearance of matter or *vice versa*. This is one of the most important of the results of the theory of relativity and may be deduced without effort from its formulæ.

Let us assume that a star that contains the energy E_0 sends out a ray of light at a certain time symmetrically in opposite directions. Let this ray have the amount of energy L, so that the energy of radiation $\frac{L}{2}$ is emitted in each direction. We shall use E_0 to denote the energy of the star with respect to a stationary system, and H_0 that with respect to a system moving with a velocity "v" relatively to the first system. Let the frequency of the ray of light referred to the resting system be ν, and the amplitude be A, then, by Maxwell's theory, the energy of the radiation must be equal to $\frac{1}{8\pi} \cdot A^2$. Now we cannot at once calculate the energy-density in the moving system by calculating

the value A, according to the formulæ of Einstein, with respect to this system. For the energy-density depends on the volume, and the influence of the formulæ of transformation on the expression for the volume has to be taken into account. Thus we must calculate as follows: according to the new formulæ the amplitude of the wave referred to the moving system becomes

$$A' = \frac{A\left(1 - \frac{v}{c}\cos\phi\right)}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Here ϕ denotes the angle that the wave-normal makes with the line connecting the star and the observer. In passing over to the moving system the volume of a sphere "V" passes over into that of an ellipsoid "V'," and consequently

$$V' = \frac{v \cdot \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}\cos\phi} V \qquad \frac{L'}{L} = \frac{\frac{1}{8\pi} \cdot A'^2 \cdot V'}{\frac{1}{8\pi} \cdot A^2 \cdot V},$$

that is, the energy-density in the moving system is

$$L' = L \cdot \frac{1 - \frac{v}{c}\cos\phi}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

This discloses the following fact. If we call the energy of the star *after* the emission of the light E_1, then

$$E_1 = E_0 - \left[\frac{L}{2} + \frac{L}{2}\right] \text{ or } E_0 = E_1 + \left[\frac{L}{2} + \frac{L}{2}\right].$$

But in the moving system

$$H_0 = H_1 + \left[\frac{L}{2}\frac{1 - \frac{v}{c}\cos\phi}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{L}{2}\frac{1 + \frac{v}{c}\cos\phi}{\sqrt{1 - \frac{v^2}{c^2}}}\right].$$

Since the two rays start out symmetrically from the star, it will experience no reaction and will remain at rest. If we now form the differences $(H_0 - E_0) = K_0$ and $(H_1 - E_1) = K_1$, then, except for a constant, K_0 and K_1 are equal to the kinetic energy $\frac{1}{2}m_0v^2$ of the star both before and after the emission. And from the above relations we get

$$K_0 - K_1 = L\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right] = \frac{1}{2}\frac{L}{c^2} \cdot v^2 + \ldots$$

Thus although the star has remained at rest relatively to the initial system after the emission, the difference $K_0 - K_1$ is not equal to zero but to $\frac{1}{2}\frac{L}{c^2}v^2$. In consequence of the amount of radiation L emitted by the star, its kinetic energy has become reduced just as if its inertial mass has decreased by the amount $\frac{1}{2} \cdot \frac{L}{c^2}$. Thus the energy L has the inertial mass $\frac{L}{c^2}$.

This discovery is particularly important because it shows that the object with which we were confronted, when we arrived at the formulæ of the special theory of relativity, namely, that of adapting the formulæ of mechanics to the new principle, could not at once be solved by seeking such re-mouldings of the Newtonian equations of motion as did not change their form when subjected to a Lorentz-transformation. From the discovery that energy has inertia it followed that we had to aim at setting up a mechanics which also ascribed gravitation to it. If this were not possible then we should have to assume that when, for example, a body is electrically

charged it increases in inertial but not in gravitational mass. Then the old law of the equality of inertial and gravitational mass, which has been known as a rigorously valid law of mechanics, would be found inaccurate and no longer exactly true. A body would be able to gain or lose inertial mass without altering its weight. An attempt to re-mould mechanics which did not start by assigning gravitation to energy would thus come into conflict with this very old fact of experience. This is what prompted Einstein, in endeavouring to adapt the formulæ of mechanics to the empirical facts of the special principle of relativity, not to seek a merely formal modification of what already existed. He had in view a system of mechanics in which the equality of inertial and gravitational mass remains preserved when the discovery of the inertia of energy is taken over as a result of the special theory of relativity.

In the third lecture we shall see that such a system of mechanics actually becomes possible by setting up a general principle of relativity. This also removes the weakest point of the "special" theory of relativity, which asserts the equivalence of only such

systems as are moving uniformly and rectilinearly. But we have seen that for direct observation all motions are relative. We cannot see why a system of mechanics should not be possible which asserts the relativity of all motions. Now, the general theory of relativity actually represents a system of mechanics in which all motions are regarded as relative, and energy has both inertia and weight, so that from the point of view of physics there is no longer a fundamental difference between energy and matter. This general theory will be treated in greater detail in the next lecture.

Now we shall answer the question why Newtonian mechanics which had not taken into account such fundamental facts of experience yet arrived at such excellent practical results. The transformation formulæ themselves furnish the answer. Since the motions which we perceive on the earth are extremely slow in comparison with the velocity of light, the omission of the terms involving $\frac{v^2}{c^2}$ and $\frac{v}{c^2}$ in the transformation formulæ of the principle of relativity was an excellent approxi-

mation to the true conditions. In principle, however, the formulæ had to take into account the fact that the velocity of light is finite and plays the unique role of a universal constant. At the same time, however, it plays the part of an upper limiting velocity. For we see from the new transformation formulæ that they become devoid of sense when $v = c$. Hence no effect can be propagated with a velocity greater than that of light. This is a result which at first gave rise to violent opposition because it was believed that in physics, too, we had the right of interpolating freely, without limits. We see, however, from the case of the velocity of light that the limits of our physical conceptions are not, as a matter of fact, the same as those of free imagination. The quantities which we call length, volume, duration, and so forth, have a meaning only within the finite limits drawn for us by observation. The fact that the velocity of light occurs as the greatest velocity with which an effect may propagate itself in nature throws into dazzling prominence the enormous significance of radiational energy in the structure of the universe, a significance which we are as yet far from comprehending to its full extent.

THIRD LECTURE

THE GENERAL THEORY OF RELATIVITY

THE special theory of relativity did not actually remove the difficulties that had beset mechanics; rather, in some respects, it added others. We shall see that only electrodynamics derived advantages from it. For the special theory of relativity asserts the following: It is true not only of motion but of all events which we perceive on the earth that they do not permit us to make evident the state of motion of the earth, which is, to a sufficient degree of approximation, rectilinear. This is tantamount to saying that all uniform rectilinear motions allow us to make assertions only about relative motions. There is no sense in deciding that the one body is at rest whilst the other is in motion, but rather we can determine only that each body is moving with

respect to the other with a certain velocity. What distinguishes the special theory of relativity from that of Galilei and Newton, which also demands that uniform rectilinear motions be relative, is that the events between two such equivalent systems are transformed by formulæ that are different from those of Newton and that take into account the constancy of the velocity of light as a newly discovered fact of experience. The discovery that energy has inertial mass was a particularly important result of the new theory.

From the point of view of the special theory of relativity all experiments on the propagation of light become intelligible, the experiment of Michelson and Morley, as well as those of Fizeau, Arago, and Hoek. Moreover, the discoveries which were made concerning moving electrons in recent years, which could not be explained earlier without the introduction of new conceptions lost their exceptional character. What used to be called apparent electromagnetic mass was recognised as the inertia of matter. For the following fact had been observed. When electrons are produced in a cathode ray tube

and traverse the space in front of the cathode with a definite velocity, then it requires a definite force to deflect them out of their direction of motion. But it is inferred from other facts that electrons cannot have material carriers but are pure energy particles. From this it was deduced that there is an analogy between such moving energy particles and moving matter, and the conception of electromagnetic mass was created. But originally it was believed that this was actually no more than an analogy. Only when the inertia of energy was discovered did these motional events become subject to the laws of mechanics.

Newtonian mechanics, however, gained nothing from these new discoveries. On the contrary, it lost one support after another. For the fact that the recognition of the inertia of energy was not associated with the gravitation of energy transgressed the law of the equality of inertial and gravitational mass, which was so surely founded on experience. It seemed possible that a body might gain in inertial mass, whereas the special theory of relativity in no wise gave us the right to assert that it would simultaneously gain in

weight. Consequently, a contradiction to experience occurred.

It was about the year 1910 that Einstein began to set up the general theory of relativity. He did so from a desire to adapt mechanics to the new discoveries and from the conviction that this could be achieved only by reforming mechanics by making the relativity of all motions its leading principle.

After the special theory of relativity of Einstein had become generally known, and after a great number of physicists had joined in working it out, many thought that the fundamentally important point was to adapt the formulæ of Newtonian mechanics to the new Lorentz-Einstein transformation formulæ. Einstein, however, recognized at once that the central point did not lie there but that it was given by the inertia of energy, and it was from this point that he attacked the problem. He made the principle of the equality of inertial and gravitational mass the starting-point of the new mechanics. Now, postulating this equality does not dispose of the question ; rather it is necessary to abstract what is essential out of this fact of experience, that which makes it possible to use it as a

foundation for the new mechanics. We must, therefore, enter more fully into this point.

The equality of inertial and gravitational mass has the following significance as regards the question of the relativity of motions. It was impossible for Newton to assert that rotations are relative, as centrifugal forces are called up by them, and as these can occur only as a result of the inertia of matter—according to his theory. He could not imagine how centrifugal forces could manifest themselves if the earth were at rest and the bodies of the universe were circling round it. But, actually, centrifugal force cannot be distinguished from a gravitational phenomenon. Since the inertial and gravitational masses are exactly equal in the case of all bodies we have no means of separating the gravitational force that we ascertain on the earth from the force that we call centrifugal. On the earth's surface we observe only the superposition of two differently directed forces; such as are directed towards the centre of the earth and such as act parallel to the meridians of latitude. Both are conditioned by that property of matter which we call mass, that is, by the property of

manifesting gravitation and inertia. Hence the view is quite permissible that this second kind of gravitation, which we call centrifugal force, is due to the gravitational action of the bodies of the universe, which revolve around the earth. Hence we see that the possibility of regarding rotations as relative motions is given by the equality of inertial and gravitational mass for all bodies.

Now, this equality of inertial and gravitational mass brings it about that all bodies on the earth's surface fall with the same acceleration. We never fail to be surprised to find that a feather and a leaden bullet fall to earth with the same speed provided the resistance of the air and other influences have been removed. Consequently, gravitational effects are recognizable only as states of acceleration. We observe that the bodies on the earth move with acceleration with respect to each other, and from this we infer that some force is involved. We observe only the accelerations. The conception of the force of gravitation which we adduce to make clear these processes is only an auxiliary conception. If we place ourselves on a spring balance in a lift, and if the pointer indicates

140 lbs., then the pointer will indicate a higher figure so soon as the lift moves upwards, just as if the passenger had increased in weight. If the lift were to move downwards, the passenger would become apparently lighter. Now, we know that there is no reason for supposing that the passenger has become heavier or lighter. We merely see that the fact of acceleration presents itself to us only in the form of a pure gravitational effect. If we were able to allow a person to fall freely in a lift so that he could make observations during a finite span of time, then objects would no longer appear to him to have weight, and his stick, for example, would remain poised in the air. Thus it is a fundamental fact of experience that there is no essential difference between accelerational and gravitational states. If an astronomer observes that some star or planet is moving with acceleration, he infers the existence of some body that produces this acceleration. Thus he at once identifies this acceleration with a gravitational effect.

Consequently, if we set up the principle of the relativity of all motions and strive to erect a system of mechanics that contains

only statements about the relative motions of bodies—and this we desire to do, because we know that only such a system of mechanics makes it possible for us to link up directly with true observation—we must at the same time arrive at a theory of gravitation, that is, a theory of accelerated motions.

A general theory of relativity demands physical laws that preserve their form independently of the system from which any event may be observed. For if we observe an occurence from the earth, we must assume that the state of motion that the earth happens to have at the moment has no bearing on the course of the occurrence, but rather that we must be in a position to find such physical laws as preserve their form independently of whether the earth be regarded as at rest and the observed occurrence as in motion, or *vice versa*. Since, when we are in a railway carriage, we are not able to determine whether our own train is moving uniformly and rectilinearly or whether the neighbouring train is so moving, as we can perceive only relative motion, the laws of mechanics must have the property of not altering their form whether we observe the event from the one

train or from the other. This, too, was the postulate of the principle of relativity of Galilei and Newton. The general principle of relativity extends this postulate to arbitrary motions, so as to give expression to the relativity of *all* motions. This does not, of course, mean that the phenomena appear in the same form to all observers. For example, if we suppose an observer in the train to let a stone drop out of the window, then the stone will fall on the embankment, and the observer, judging from his observation, will assert that the stone has fallen to the earth rectilinearly. Thus the trajectory of the stone is a straight line for the observer in the train. But this is not the case for an observer standing on the embankment. To the latter the stone will appear to move in a parabolic orbit towards the earth. It is clear from this that the forms of the phenomena may be very different. The assertion concerning the form of the orbital curve has no absolute meaning. But the law that allows this event to be understood and mastered must be able to be formulated independently of the situation of the observer, and hence must preserve its form if we replace

in it the co-ordinates of space and time by such as are used by a second observer, moving arbitrarily with respect to the former, to describe the event. This is the postulate of the general covariance of physical laws as contained in the general theory of relativity.

But in putting this postulate into practice a difficulty is encountered which initially raised the gravest scruples. It became evident that a system of mechanics that demanded the relativity of *all* motions entailed the giving up of the laws of Euclidean geometry. For physical laws that enlist the help of the rules of analytical geometry for their formulation cannot preserve their form in the transition to a system that is moving arbitrarily with respect to the first system. Consequently there must be hypotheses in the laws of Euclidean geometry that stand in contradiction with the hypotheses of the relativity of all motions. We must therefore make a short digression into the realm of geometry.

Euclidean geometry, being built up on a few axioms, treats exclusively of configurations that are gained by abstractions from

experience. Consequently its theorems have nothing to do directly with experience. But from the moment when we apply its laws to the material world, to measure distances or volumes, the axioms become hypotheses of physics and we must then ask ourselves whether these hypotheses are in agreement with experience. A fundamental assumption of Euclidean geometry is that when finite rigid configurations move in space they do not alter their dimensions. The possibility of proving the congruence of two given triangles rests on just that fact. To prove that, say, the triangle ABC is congruent with the triangle A′B′C′ we apply one to the other and, in doing so, we assume that no change of dimension takes place during the process up to its completion when coincidence occurs. On a plane or on a spherical surface we can, as a matter of fact, slide a triangle over the whole surface without an alteration of dimensions occurring. This would not be possible on an oval-shaped surface. If we slide a triangle along on such an oval surface, which is irregularly curved, the triangle changes, and we have no means of setting up theorems of congruence in the way in

which such theorems occur in Euclidean geometry. If, for example, we further assume that this oval surface is made of rubber which has been blown up by means of a gas, and if we alter the pressure within the oval, then its form changes from moment to moment, so that we cannot assert that a particular constant geometry holds on it. Rather this will depend on the pressure of the gas within the oval surface. Corresponding results hold in three-dimensional space but do not allow of illustration in the same simple way.

Thus the question arises: Does the material world, as regards its geometric conditions, behave analogously to the sphere or to the oval surface, that is, is it possible to move bodies of finite size about in the world without causing them to alter their dimensions? The answer depends on whether we are justified in using for the distance between two points with the co-ordinates x_1, y_1, z_1, and x_2, y_2, z_2, respectively, the following expression which follows from Pythagoras' theorem:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

and in using for the line-element " ds " in the

case when the two points become indefinitely close, the expression

$$\sqrt{dx^2 + dy^2 + dz^2}.$$

If we use this expression for the distance between two points in space in mechanics we make the hypothesis that finite rigid bodies may be moved freely in space without altering their dimensions.

The principle of the relativity of all motions requires the immutability of physical laws with regard to arbitrary transformations, since all the systems of reference are equivalent for the co-ordinate measurements. The Euclidean expression for the distance between two points, however, alters its form if in place of x, y, z we write any other arbitrary co-ordinates, unless the new co-ordinates correspond to the transition to a co-ordinate system which is moving uniformly and *rectilinearly* with respect to the initial system. Hence, in striving to obtain laws that preserve their form when any transformation is carried out we must give up the postulate that the Euclidean measure-determination be valid; otherwise there is no relativity of *all* motions.

Now, in the year 1854 Riemann stated a

discovery the significance of which for physics was not understood for nearly half a century. He asserted that the geometrical laws in the material world are not given to us from the outset as a rigid structure into which we place the physical world. Rather it is a problem of physics to discover the laws which we may claim as the geometrical laws of the material world. The form of the expression for the distance between two points is determined by the measure-conditions of the manifold that we call space. If these conditions correspond to those that exist on the surface of a sphere or plane, then Pythagoras' theorem holds and the distance between two points may be measured by means of the expression :

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

and the line element by the expression

$$ds = \sqrt{dx^2 + dy^2 + dz^2}.$$

But if we have to deal with a configuration of the type of the oval surface in the two-dimensional space, then the distance between two points is by no means given by so simple an expression in the co-ordinates. Whether a manifold of space-points behave like a plane configuration or not depends on whether

its curvature has the same value at each point. "Curvature" here denotes in general an expression in the co-ordinates, the value of which is independent of the particular choice of the co-ordinates and which measures the deviation of the configuration from a "plane" Euclidean configuration. If the curvature is nil at every point, then the configuration behaves like a plane, or a three- or multi-dimensional space, respectively. If it is positive but constant, then we are dealing with a sphere or with what corresponds to it in multi-dimensional space. If it varies from point to point, we are dealing with a manifold corresponding to that represented in the world of surfaces by the oval. Thus Riemann created the mathematical tools necessary for the foundation of the relationships underlying the conditions of space. Riemann had, however, only a faint presentiment of how we were to arrive physically at the actual discovery of the geometric measure-conditions in Nature. Yet, clearly recognizing the problem presented, he emphasized at the conclusion of his essay that only by further developing mechanics should we get into a position to determine whether we are justified in using

Euclidean geometry in the world of material bodies or not, and that the inner ground of the measure-conditions are to be sought in certain forces whose discovery was one of the most important problems of physics. Up till Einstein's researches were made known no one had made even the vaguest conjecture as to what sort of force it was to which we had to assign the special function in the world of governing the geometric conditions in it. There can, of course, be no doubt that it plays a fundamental part in the edifice of the world.

To grasp the nature of such an effect let us consider the following example: A physicist undertakes the task of measuring out the surface of a table by covering it with very small iron rods and counting the number of squares. Further, we shall assume that the table is round. Now he finds to his astonishment that the circumference of the table, divided by its radius, does not give the figure 2π. Perhaps he would never succeed in understanding this phenomenon if he did not learn from other indications that the table has different temperatures at different points. Suppose the temperature to be greater at

the edge than in the middle. Then the rods expand and we cannot expect that the circumference divided by the radius will give us 2π. If the same observer had used ivory or quartz rods he would have found no departure from Euclidean geometry. Now if we were to conceive the idea of interpreting thermal conditions of the world by means of the geometric conditions, we should arrive at an enormously complicated picture of the conditions, since the influence of heat on the conditions of expansion is different for each body, and nature has made it easy for us to measure the dependence of the length of an object on its temperature and to take it into account.

But let us assume that there is a natural phenomenon which has the property of exhibiting no differentiation of behaviour towards substances, that is, such that it influences all bodies in the same way independently of whether it is made of metal or wood. For this phenomenon the specific expansion would be the same for all bodies and the physicist who now finds a departure from the Euclidean geometry of measurement on his table would have no means, such as testing by rods, that

are free from this influence, of showing that he would furnish full explanations with Euclidean geometry as soon as he took into account the specific expansion of bodies. If there were such a phenomenon on the earth then nothing would remain for us but to regard such departures from Euclidean measure-relations as expressions of its action.

If we formulate the problem in this way then it seems very improbable that a phenomenon of such a special type should exist on the earth. But it is just this discovery that Einstein disclosed to us in pointing out that gravitation has this very particular property of acting in the same way on all substances. The property that distinguishes gravitation from all other effects is that under the influence of the earth's gravitation all bodies fall to the earth with exactly the same acceleration, being another form of the phenomenon that inertial and gravitational mass are the same for all bodies. This gives us the possibility of proclaiming gravitation as the author of the geometric conditions. The only question that remains to be decided is whether this point of view is fruitful, that is, whether it leads to a simple theory of gravita-

tional phenomena. Along this line of argument Einstein has actually succeeded in combining and solving three hitherto distinct problems, namely, that suggested by Riemann concerning the inner ground of the measure-conditions of space, the problem of the relativity of all motions, and the problem of the gravitational action of all bodies on one another, a problem which had remained unsolved for two centuries. Einstein's theory of relativity solves these problems by enlarging to an incredible extent our knowledge of the significance of space and time in the laws of physics. The fundamental fact of experience on which this theory is founded is the equality of inertial and gravitational mass. This fact, not understood earlier, was applied as the principle of equivalence formulated by Einstein, with fruitful effect to the comprehension of gravitational phenomena.

As we saw earlier, the preliminary condition furnished by Nature for regarding accelerated motions and rotations as relative motions is the equality of inertial and gravitational mass. Hence it follows that if we raise it to the rank of a strictly valid physical law, every event which we observe and in which

we perceive an alteration, produced by the action of a gravitational field, would occur in exactly the same way if, under the assumption that there is no gravitation, we were to observe it from a system that is subject to the acceleration that would be characteristic of the action of the gravitational field. This is the inner meaning of the equality of inertial and gravitational mass. As we cannot distinguish a state of acceleration from a gravitational state, we may postulate the complete *equivalence* of both types of phenomena. Einstein built up his theory on the principle of equivalence as the ground hypothesis. It assures for him that all the laws of mechanics are statements concerning relative motions.

Our next concern is to set up a law of motions that describes the motions of bodies under the influence of inertia and weight and that has the peculiarity of preserving its form when we carry out the transition from one system to another that is accelerated in any way whatsoever with respect to the first. A suitable law was found in the law of inertia that occurs in Newtonian mechanics ; it had, of course, to be formulated so that it is no

law of action at a distance, and so that it involved no assumption about the existing measure-relations.

The law of inertia states that a body which is not subject to external forces moves along the straightest or shortest path. According to Newtonian mechanics the shortest path is at the same time the straight line of Euclidean geometry. Einstein made a fully corresponding law of motion the starting point of his theory: every body moves under the influence of inertia and gravity along the shortest or straightest path, that is, in stricter language, along a geodetic line of the space-time-manifold. But this is by no means in general a straight line, for the body is subject to a constraining influence, namely, that of the action of gravitation. This distorts the path. But this distortion may be regarded as an expression of geometrical measure-relations of space and time, which vary from point to point and depart from the Euclidean laws of measure. To make this clear let us consider the following case, taken from Newtonian mechanics. A sphere is rolling to and fro in a dish. If we were to apply the law of inertia to this motional event, then

this would by no means give us the true motion, for the sphere which is constrained to move in the dish cannot move rectilinearly but rolls to and fro on the inner surface. Nevertheless, in Newtonian mechanics, too, there is a law of motion for this sphere. It states that the sphere, when moving on this spherical surface of the dish, does so along a shortest path. This is a geodetic curve along the spherical shell. The new law of motion of Einstein contains the fully corresponding statement. Every body in the gravitational field moves along a geodetic line. In this case, however, the surface on which the sphere is compelled to move is not given as a visible rigid surface, but is represented by the compelling effects of gravitation, to which all bodies are subject in the same way.

This law may be formulated so that it is a law of contiguous action, that is, so that it contains only infinitely small space-time intervals, and preserves its form, independently of the co-ordinate system in which the event is observed. As a necessary supplement to this law of motion we require a rule to find from the distribution of the masses the departures from rectilinear uniform motion

which we observe in the orbits of bodies under the influence of gravitation. Here Einstein has set up a law of which the purport is somewhat as follows: The deviations from Euclidean measure-relations, as formed from point to point and as giving rise to the curvilinear accelerated motions of bodies, may be measured by an expression developed by Riemann and, in value, independent of the choice of co-ordinates. According to Einstein the value of this function is proportional to the quantities, mass and energy, which excite the gravitational field.

Thus if we contrast Newtonian mechanics with Einstein's theory, the position is as follows: Newtonian mechanics is built up on the law of inertia. This is founded on the never-to-be-observed phenomenon of the motion of a body that is not subject to external forces. In contradistinction to this the most elementary law of motion of Einstein's theory relates to the motion of a body under the influence of inertia and gravity, in the simplest case, to the motion of two bodies that act on each other in consequence of their gravitation. The equality of inertial and gravitational mass which was known even to Newton,

but which appears in his mechanics merely as a remarkable fact, becomes raised to the rank of a strictly valid physical law and allows us to assert the relativity of all motions directly from the equivalence of accelerational and gravitational states. Thus the occurrence of centrifugal forces on the surface of the earth is declared to be a consequence of the relative motions of bodies with respect to the earth. Whether we assume that the earth is at rest and the bodies circle round the earth, or that the bodies are at rest and the earth is rotating on its axis, is fundamentally unessential in describing the whole event. The occurrence of centrifugal forces is a consequence of the mutual mass attraction. This stand-point is possible because inertial and gravitational mass are always equal to each other.

In place of Newton's law: *force* equals *mass* times *acceleration*, there now occurs in the general theory of relativity the law which regulates the relationship of the measure-conditions through the quantities mass and energy that excite the gravitational field. And this law makes it possible, when these two quantities are known, to determine the

special form of the geodetic line as the orbital curve in the space-time-manifold.

The two theories seem to have almost nothing in common with each other. The only way to decide their utility is, in addition to proving the absence of inner contradictions, to test their laws by experiment. This brings to light the following surprising result. If, without introducing supplementary hypotheses or other constants except that of gravitation, we calculate the orbit of a point-mass in the gravitational field of the sun according to the formulæ of Einstein's mechanics, we again get, to a first approximation, for the orbital curves of the planets the ordinary Kepler orbits. But, nevertheless, small deviations from the results of the Newtonian theory manifest themselves, and these allow us to decide which of the two theories is the more powerful. Thus, according to the Newtonian theory, Kepler's planetary ellipses were to be at rest relatively to the stellar system so long as only the attraction of the sun was taken into account. On the other hand, according to Einstein's theory, when a body moves along the straightest path under the influence of the sun's attraction,

it describes an ellipse, it is true, but this ellipse slowly revolves, and the amount of this turning is, for a planet at the distance of Mercury, about 43 seconds of arc per century. And this is approximately the amount which Leverrier had calculated as early as 1850 from observations of Mercury, and which had hitherto remained inexplicable. Thus the only discrepancy that occurs between calculation and observation, according to Newton's theory, in the planetary system is a first test in favour of the new mechanics, and shows its superiority over that of Newton. Unfortunately there is no possibility of investigating the corresponding effect in the case of the planet Venus. The eccentricity of its orbit is so small that we cannot with certainty determine the rotation of the major axis of its orbit. In the case of all other planets the amount of this difference between the two theories is too small to allow us to obtain it at present from observation.

But there are still further possibilities of testing the theory of relativity, and they are the following: According to the hypothesis of equivalence which asserts a complete equivalence of accelerational and gravi-

tational states, the inertial mass is always equal to the gravitational mass. Since, even according to the special theory of relativity, energy has inertial mass, it must also have gravitational mass, and a ray of light that passes in close vicinity to the sun is consequently subject to the gravitational action of the sun. A result of this effect is that if we photograph a star in the immediate neighbourhood of the sun during a solar eclipse, then this star appears displaced from the position which it usually occupies when we photograph this region of the heavens in the absence of the sun. For, according to the new theory, the energy which the ray of light represents has gravitation, and is consequently subject to the gravitational influence of the sun just like a comet that flies past the sun at the same distance with the velocity of light. The ray is deflected from its straight path and reaches the observer, who directs his telescope in a straight line at it, from a direction different from what it is when the motion is entirely along a straight line. This effect is great enough to be measured. In view of the eclipse of the sun on 29th May, 1919, England had equipped two expeditions to

investigate this question, and the result was that the stars in the vicinity of the sun were found to be displaced by the amount and in the direction predicted by Einstein. In later eclipses it will have to be tested whether all errors that might give this effect spuriously have been eliminated. With the present state of affairs we have the right to assume that the gravitation of light has actually been proved to exist, with a degree of certainty, by the English expeditions.

A third possible proof is the following: It has a special significance in that it offers an experimental test of Einstein's theory without assuring the validity of the special form of his laws, but only the principle of the relativity of motions. If this deduction were not to be verified then the fundamental assumptions of the theory of relativity could no longer be maintained.

We consider a simple optical process, namely that a light ray falls from the ceiling of the observation chamber to the floor. On the ground of the principle of equivalence, Einstein's theory asserts that whether this optical event be observed in the gravitational field of the earth or in relation to a system of refer-

ence that is accelerated by 32 feet per second upwards is a matter of indifference so far as the appearance of the event is concerned. Let us, therefore, calculate the influence of the acceleration of the system of reference on this optical event. Let us assume that at the moment at which the ray leaves the source of light, all was at rest; then, in consequence of the acceleration of 32 feet per second the observer will have attained a velocity of $\frac{32 \cdot h}{c}$ at the moment the ray reaches him; here "h" is the distance of the source from the observer (in feet), and c is the velocity of light (in feet per sec.). The observer, therefore, perceives a Doppler effect, that is, the frequency of the light, which we suppose to be γ at the source, will appear at his point of observation to be $\gamma \cdot \left(1 + \frac{32 \cdot h}{c^2}\right)$. According to the principle of correspondence this effect must also present itself when the observer follows this process in a gravitational field that is characterized by the gravitational acceleration of 32 feet per second per second. From this reflection it follows generally that if an observer perceives a source of light that is situated at a point where the

potential of the gravitational field is higher or lower than at the point of observation, then the light from this source will show an apparent Doppler effect. If we calculate this effect for the sun, then we get a displacement of the Fraunhofer lines to the extent of $2 \cdot 10^{-6}$ times the wave-length towards the red end of the spectrum. This denotes a shift of 0·008 Å for a wave-length $\lambda = 400 \mu\mu$. The proof of this displacement is at the present time the most important for Einstein's theory. For if this effect is found not to exist then the principle of equivalence and that of the relativity of all motions may no longer be maintained. Up to the present it has not been proved with certainty that this effect does not exist. Rather, we have even now important indications that it is present. But it will still take many years before we shall be able to assert with certainty that it *does* exist.

Moreover this test of the shift of the spectral lines has yet another great significance. From it methods may be developed that will at once become of the greatest importance to astronomy, as they will give us the possibility of finding from spectral observations of stars with

what masses we are dealing. It will give astronomers the means of investigating the distribution of the masses in the universe. There are, as a matter of fact, indications that the particularly great stars, of which it is assured that they are greater than the sun, exhibit a displacement of the spectral lines. The results are not yet definitely certain, since we have so far no other means of testing whether we are actually dealing with such great masses. But it opens up great and important avenues for exploring the gravitational conditions of the universe.

If the theory were to be confirmed without exception then we should arrive at results that would differ fundamentally from our earlier results. We should no longer be able simply to take over Euclidean geometry into the formulæ of physics, as something that is in a certain sense given from the outset, like pigeon-holes into which the physical events are parcelled, but the study of geometrical conditions in nature would become a new problem of physics. These are relationships of incredible consequence. Furthermore, we should have the certainty that inertial and gravitational mass are equal, and that energy

does not differ in character from matter. For we shall be able to regard the inertial mass of matter as the inertia of enormous amounts of energy, which the bodies have stored up in themselves. The laws of conservation of energy and mass which were hitherto unconnected in physics would become fused into a law embodying the principle of conservation of energy.

On account of its great mathematical simplicity Newtonian mechanics will never lose its great practical importance. But we must remain unprejudiced enough to recognize that it contains assumptions that are apparently not admissible in physics.

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